Benha University
Faculty of Engineering- Shoubra
Eng. Mathematics & Physics Department
Qualifying Studies (Mathematics)

Final Term Exam
Date: May , 2016
Operations Research EMM 406
Duration: 3 hours

- Answer **All** questions The exam consists of one page
- No. of questions: 4 Total Mark: 200

Question 1

(a)Write the mathematical form of mathematical programming problem. Also, classify the mathematical programming problems. **20**

(b)Write and solve the dual problem of the LP problem: **30**

$$\text{minimize } f = 3x + y$$

$$\text{s.t } \quad x - y \le 4, \quad -x + y \le 1, \quad x + y \ge 3, \quad x, y \ge 0$$

Question 2

Solve the LP problems:

(a) maximize $f = x - 3y + 3z$ **25**

$$\text{s.t } \quad 2x + y - z \le 4, \quad 4x - 3y \le 2, \quad -3x + 2y + z \le 3, \quad x, y, z \ge 0$$

(b) minimize $f = -5x - y - 4z$ **25**

$$\text{s.t } \quad x + y + 2z \le 20, \quad 2x + 3y + 2z = 10, \quad x + 2y + 2z \ge 6, \quad x, y, z \ge 0$$

Question 3

(a) State the definition of convex set. **5**

(b) State the definition of concave function. **5**

(c)Prove that: For any convex programming problem, the set of all optimal solutions is convex. **20**

Question 4

(a)A manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop1, which performs the basic assembly operation, must work 5 man-days on each truck but only 2 man-days on each automobile. Shop 2, which performs finishing operations, must work 3 man-days on each automobile or truck that it produces. Because of men and machine limitation shop1 has 180 man-days per week available while shop 2 has 135 man-days per week. If the manufacturer makes a profit of LE 300 on each truck and LE 200 on each automobile. How many of each should he produce to maximize his profit? **30**

(b)Solve the assignment problem: (c)Solve the transportation problem **40**

Machine

$$\text{Job} \quad \begin{bmatrix} 4 & 8 & 12 & 6 \\ 18 & 7 & 10 & 9 \\ 8 & 5 & 11 & 7 \\ 16 & 7 & 8 & 5 \end{bmatrix}$$

					Supply
	3	0	5	4	15
	1	3	5	0	20
	6	2	4	5	25
Demand	10	15	20	15	60

Good Luck *Examiners: Dr. Mohamed Eid* *Dr. Fathi Abd-Elsallam* *Dr. Zaki Ahmed*

Model Answer

Question 1

(a) A mathematical programming problem can be formulated as follows:

maximize (or minimize) $f(x)$

subject to $M = \{x \in R^n : g_r(x) \leq 0,\ r = 1,2,\dots,m\}$

f (x) is called the objective function

x is the vector of the variables (decision variables, unknowns)

$g_r(x) \leq 0,\ r = 1,2,\dots,m$ are constraints.

M is called the feasible domain of the problem which is formed by the constraints.

The mathematical programming problems can be classified as:

1- Linear programming (LP) problems when f(x) and $g_r(x) \leq 0,\ r = 1,2,\dots,m$ are linear functions. It take the form

Maximize (or minimize) f(x) = $c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to
$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$
$$x_1, x_2, \dots, x_n \geq 0.$$

This problem can be written in matrix form as follows:

Maximize (or minimize) f(x) = C x

s.t $\quad Ax \leq B, \quad x \geq 0$

2- Non linear programming problems if f(x) is non linear or either one from $g_r(x) \leq 0,\ r = 1,2,\dots,m$ is non linear function.

3- Quadratic programming problems if f(x) is quadratic function and $g_r(x) \leq 0,\ r = 1,2,\dots,m$ are linear functions.

4- Integer programming problems if the decision variables are integers.

5- Mixed integer programming problems if some of the decision variables are integers.

---20 *Marks*

(b)The dual problem is:

Maximize $g(y) = 4y_1 + y_2 + 3y_3$
$$y_1 - y_2 + y_3 \leq 3$$
$$-y_1 + y_2 + y_3 \leq 1, \quad y_1 \leq 0, \quad y_2 \leq 0, \quad y_3 \geq 0$$

Then: Maximize $g(y) = -4y`_1 - y`_2 + 3y_3$

s.t $\quad -y`_1 + y`_2 + y_3 \leq 3$

$y`_1 - y`_2 + y_3 \leq 1, \quad y`_1 \geq 0, \quad y`_2 \geq 0, \quad y_3 \geq 0$

The steps of the simplex method goes as:

B.V	$y`_1$	$y`_2$	y_3	s_1	s_2	Solu
s_1	– 1	1	1	1	0	3
s_2	1	– 1	1	0	1	1
f	4	1	–3	0	0	0
s_1	– 2	2	0	1	– 1	2
y_3	1	– 1	1	0	1	1
f	7	– 2	0	0	0	3
$y`_2$	– 1	1	0	1	– 1/2	1
y_3	0	0	1	0	1/2	2
f	5	0	0	1	2	5

Then the optimal value is $g^* = f^* = 5$ and the optimal solution (x, y) = (1, 2).

---*30 Marks*

Question 2

(a)The standard form of this problem is:

maximize f = x – 3y + 3z

s.t $\quad 2x + y - z + s_1 = 4$

$4x - 3y + s_2 = 2$

$-3x + 2y + z + s_3 = 3, \quad x, y, z, s_1, s_2, s_3 \geq 0$

where s_1, s_2 and s_3 are slack variables.

The steps of the simplex method goes as:

B.V	x	y	z	s_1	s_2	s_3	Solu
s_1	2	1	–1	1	0	0	4
s_2	4	–3	0	0	1	0	2
s_3	–3	2	1	0	0	1	3
f	–1	3	–3	0	0	0	0
s_1	–1	3	0	1	0	1	7
s_2	4	–3	0	0	1	0	2
z	–3	2	1	0	0	1	3
f	–10	9	0	1	0	3	9
s_1	0	9/4	0	1	1/4	1	15/2
x	1	–3/4	0	0	1/4	0	1/2
z	0	–1/4	1	0	3/4	1	9/2
f	0	3/2	0	0	5/2	3	14

This is the optimum case. Then, the optimal solution is:
$(x^*, y^*, z^*) = (1/2, 0, 9/2)$ and the optimal value is $f^* = 14$.
---*25 Marks*

(b) The standard form of this problem is:

minimize $f = -5x - y - 4z$

s.t. $x + y + 2z + s_1 = 20$

$2x + 3y + 2z + u = 10$

$x + 2y + 2z - t + v = 6$, $\quad x, y, z, s_1, t, u, v \geq 0$

where s_1 is slack variable, t is surplus variable and u, v are artificial variables.

Let w = u + v. Then, the objective of phase one is:

$w + 3x + 5y + 4z - t = 16$

The steps of phase one goes as table :

B.V	x	y	z	t	s_1	u	v	Solu
s_1	1	1	2	0	1	0	0	20
u	2	3	2	0	0	1	0	10
v	1	2	2	-1	0	0	1	6
f	5	1	4	0	0	0	0	0
w	3	5	4	-1	0	0	0	16
s_1	1/2	0	1	1/2	1	0	–1/2	17
u	1/2	0	-1	3/2	0	1	–3/2	1
y	1/2	1	1	–1/2	0	0	1/2	3
f	–9/2	0	3	1/2	0	0	–1/2	–3
w	1/2	0	-1	3/2	0	0	–5/2	1
s_1	1/3	0	4/3	0	1	–1/3	0	50/3
t	1/3	0	– 2/3	1	0	2/3	-1	2/3
y	2/3	1	2/3	0	0	1/3	0	10/3
f	–13/3	0	–10/3	0	0	1/3	0	– 10/3
w	0	0	0	0	0	-1	-1	0

This is the end of phase one. Also, it is the of phase two.
This is the optimum case. Then, the optimal solution is:
$(x^*, y^*, z^*) = (0, 10/3, 0)$ with optimal value $f^* = -10/3$.
---*30 Marks*

Question 3

(a)**Convex set**: A set M is called convex if for all two points x and y in M, all points of the line segment $\lambda x + (1-\lambda)y$, $0 < \lambda < 1$, lie in M.
---*5 Marks*

(b)**Concave function**: A function f is called concave on a set M if for all two points x and y in M, $f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda)f(y), \quad 0 < \lambda < 1$

--*5 Marks*

(c)Theorem:

--*20 Marks*

Question 4

(a)Let x be the number of trucks and y the number of automobiles to be produced per week. Then

	x	y	available
Shop 1	5	2	180
Shop 2	3	3	135

Then the constraints are:

$5x + 2y \leq 180,$

$3x + 3y \leq 135.$

The objective function is:

$f = 300x + 200y.$

Then the LP model is:

maximize $f = 300x + 200y$

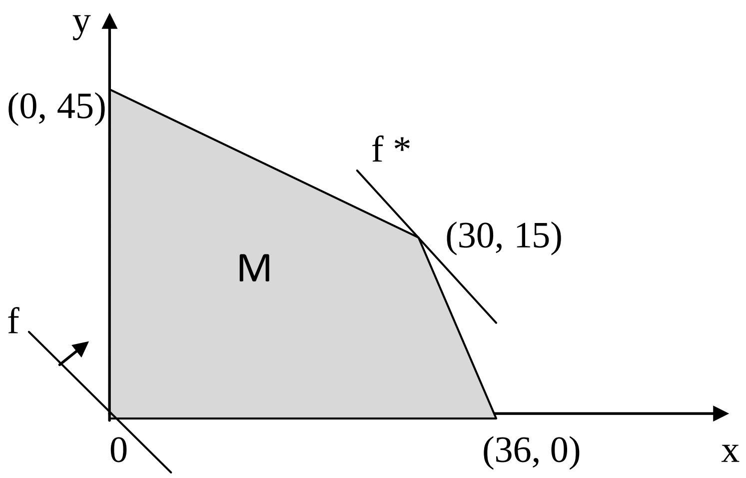
s.t $5x + 2y \leq 180$

$3x + 3y \leq 135,$

$x, y \geq 0.$

This LP problem can be solved graphically as shown in Figure 8. Then the maximum profit is LE 12000 at the optimal solution

$(x^*, y^*) = (30,15).$



--*30 Marks*

Dr. Mohamed Eid

Final Exam and ILOs

Course Title: Operations Research Code: EMM 406

Questions	ILOs				
	Knowledge and Understanding		Intellectual Skills		Professional and Practical Skills
	2.1.1	2.1.2	2.2.3	2.2.7	2.3.2
Q1	√		√		
Q2			√	√	
Q3	√		√		
Q4		√	√	√	√

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